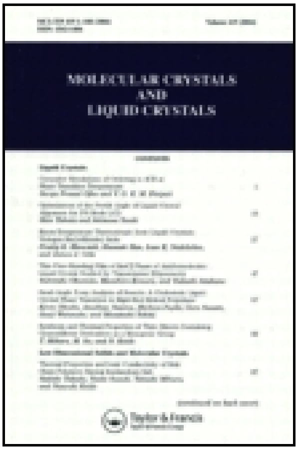

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F. M. Leslie [a] & C. M. Waters [b]

[a] Strathclyde University, Mathematics Department, Glasgow, G7 1XH, UK

[b] Royal Signals and Radar Establishment, Malvern, Worcestershire, WR14 3PS, UK

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Light Scattering From a Nematic Liquid Crystal in the Presence of an Electric Field†

F. M. LESLIE
Mathematics Department, Strathclyde University, Glasgow G1 1XH, UK

and

C. M. WATERS
Royal Signals and Radar Establishment, Malvern, Worcestershire WR14 3PS, UK

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The intensity and spectrum of light scattered by a nematic liquid crystal of positive dielectric anisotropy (5CB) in the presence of an *ac* squarewave electric field have been measured. The results for the twist-bend mode agree well with the theory developed by the Orsay Group and lead to satisfactory values for elastic and viscous coefficients. The results for the splay-bend mode, however, do not, showing a strong dependence on the frequency of the applied field, with scattering no longer of Lorentzian form.

In an attempt to explain the above we examine appropriate solutions of the continuum equations including both conduction and flexoelectric effects. In line with our experimental results, it turns out that neither of these complications affects the twist-bend mode, but that both contribute to the splay-bend mode, although the influence of flexoelectricity is likely to be small. However, conduction can significantly alter the splay-bend mode, there being the possibility of a more complex behaviour when this additional effect is included, which appears to explain the observed non-Lorentzian response.

INTRODUCTION

In a nematic liquid crystal the long range orientational order undergoes thermally driven fluctuations about an equilibrium position to

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produce an intense Rayleigh scattering of light. Many investigators have studied this quasi-elastic light scattering by nematics using photon correlation techniques to determine the intensity and spectrum of scattered laser light. The theory of such light scattering from nematic liquid crystals has been developed by de Gennes[1] and the Orsay Group[2] using the Ericksen-Leslie continuum equations to describe the overdamped fluctuations. This provides a possible means of determining the ratio of elastic constants from intensity measurements as a function of the scattering wave-vector (scattering angle), and also viscoelastic ratios from measurement of the band-width of the spectrum of the scattered light. General accounts of this topic and further references are available in the books by de Gennes[3] and Chandrasekhar.[4]

As de Gennes[1] discusses the application of a stabilising magnetic field damps the amplitude of the alignment fluctuations, and moreover allows the determination of individual viscous and elastic coefficients from measurements of the damping or decay rate of these fluctuations as a function of the applied field. Martinand and Durand[5] measured the bend elastic constant in this way for MBBA (4′-methoxybenzylidene-4-n-butylaniline), but using an electric field applied across a nematic cell in which the alignment is planar, since the dielectric anisotropy is negative for this material.

In this paper we study experimentally the quenching of the alignment fluctuations in the nematic 5CB (4-cyano-4′n-pentylbiphenyl) by an *ac* squarewave electric field. Since this nematic has positive dielectric anisotropy, the voltage is applied to a cell containing the liquid crystal in the homeotropic configuration. As the Orsay Group[2] show, the fluctuations in general separate into two distinct modes, one called the splay-bend mode and the other the twist-bend mode, and with normal incidence and homeotropic alignment it is possible to examine each mode independently by a suitable choice of the polarisation of the incident and scattered light (see Figure 1). In both cases the scattered light is analysed in the scattering plane, while the incident polarisation is in this plane for the splay-bend mode, but perpendicular to this plane for the twist-bend mode.

Our experimental results for the twist-bend mode are as predicted by the Orsay Group and lead to values for both the twist elastic and viscous coefficients in good agreement with previous measurements by other methods. However, our results for the splay-bend mode do not agree satisfactorily with existing theory, and in the latter part of the paper we attempt an explanation of this discrepancy by providing a more detailed analysis of fluctuations in an electric field.

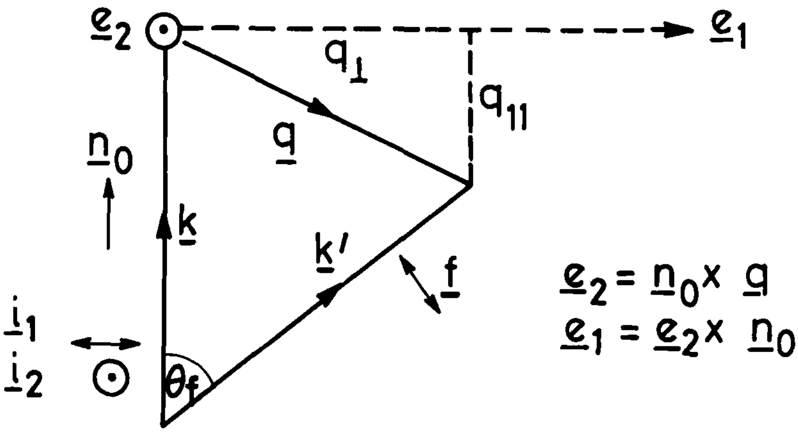


FIGURE 1 Scattering vector diagram showing relative orientations of vectors representing the undisturbed director $\mathbf{n}^0$, the scattering vector $\mathbf{q}$, the polarisation of the scattered light $\mathbf{f}$, and the polarisation of the incident light chosen to select the splay-bend mode $\mathbf{i}_1$, or the twist-bend mode $\mathbf{i}_2$. θ_f is the scattering angle.

2. THEORY OF FLUCTUATIONS IN AN ALIGNED NEMATIC

In this section we briefly summarise the Orsay analysis of the damping of small disturbances in a uniformly aligned nematic, their predictions being required in the ensuing sections. Our presentation follows rather closely that given by Leslie[6] and essentially employs his notations.

Let $\mathbf{n}^0$ denote the initial uniform alignment of the director, possibly produced by a parallel uniform magnetic field of strength H, and consider a small perturbation of this alignment of constant wave-vector $\mathbf{q}$ and frequency ω of the form

$$\mathbf{n} = \mathbf{n}^0 + \hat{\mathbf{n}} \exp i(\mathbf{q} \cdot \mathbf{x} - \omega t) \qquad \hat{\mathbf{n}} \cdot \mathbf{n}^0 = 0, \tag{2.1}$$

where $\hat{\mathbf{n}}$ is a constant vector, $\mathbf{x}$ denotes the position vector and t time. Associated with the above is a corresponding flow perturbation given by

$$\mathbf{v} = \hat{\mathbf{v}} \exp i(\mathbf{q} \cdot \mathbf{x} - \omega t) \qquad \hat{\mathbf{v}} \cdot \mathbf{q} = 0, \tag{2.2}$$

where $\hat{\mathbf{v}}$ is a constant vector, constrained as indicated on account of the assumed incompressibility. It is convenient to define an angle ϕ by

$$q \sin \phi = \mathbf{n}^0 \cdot \mathbf{q} \qquad q = |\mathbf{q}|, \tag{2.3}$$

and to decompose the vectors $\hat{\mathbf{n}}$ and $\hat{\mathbf{v}}$ into components as follows

$$q\,\hat{\mathbf{n}} = n_1(\mathbf{q} - q\sin\phi\mathbf{n}^0) + n_2\mathbf{n}^0\times\mathbf{q},$$

$$q\,\hat{\mathbf{v}} = v_1(q\mathbf{n}^0 - \sin\phi\,\mathbf{q}) + v_2\mathbf{n}^0\times\mathbf{q}. \qquad (2.4)$$

In this event the continuum equations with products of perturbations neglected reduce to

$$(\rho\omega + iq^2g(\phi))v_1 + iq\omega m(\phi)n_1 = 0,$$

$$iqm(\phi)v_1 + (i\omega\gamma_1 - q^2f(\phi) - \chi_aH^2)n_1 = 0, \qquad (2.5)$$

with

$$2g(\phi) = \alpha_4 + (\alpha_3 + \alpha_6)\cos^2\phi + (\alpha_5 - \alpha_2)\sin^2\phi + 2\alpha_1\sin^2\phi\cos^2\phi,$$

$$m(\phi) = \alpha_2\sin^2\phi - \alpha_3\cos^2\phi, \quad f(\phi) = K_1\cos^2\phi + K_3\sin^2\phi, \qquad (2.6)$$

and

$$(\rho\omega + iq^2k(\phi))v_2 - iq\omega\alpha_2\sin\phi n_2 = 0,$$

$$iq\alpha_2\sin\phi\, v_2 - (i\omega\gamma_1 - q^2h(\phi) - \chi_aH^2)n_2 = 0, \qquad (2.7)$$

with

$$2k(\phi) = \alpha_4 + (\alpha_5 - \alpha_2)\sin^2\phi,$$

$$h(\phi) = K_2\cos^2\phi + K_3\sin^2\phi. \qquad (2.8)$$

In the above ρ denotes density, χ_a the diamagnetic susceptibility anisotropy, and the α's and K's viscous and elastic coefficients, respectively. Apart from differences in notation and the use of the Parodi relation,[7] the above equations are identical to those obtained by the Orsay Group.[2]

From equations (2.5) and (2.7) it is immediately clear that the perturbations uncouple into two distinct modes, one involving the components in the plane of the initial alignment $\mathbf{n}^0$ and the wave-vector $\mathbf{q}$, and the other the components normal to this plane. The former is referred to as the splay-bend mode, and the latter as the twist-bend mode. For each mode the compatibility condition for the two equations leads to a quadratic for the unknown ω, regarding $\mathbf{q}$ as given. As the Orsay Group show, the two roots for the splay-bend mode are given to a good approximation by

$$\omega_1^{(1)} = -ig(\phi)(q^2 f(\phi) + \chi_a H^2)/\gamma_1 \tilde{g}(\phi),$$

$$\omega_2^{(1)} = -iq^2 \tilde{g}(\phi)/\rho, \quad (2.9)$$

where

$$\tilde{g}(\phi) = g(\phi) - m^2(\phi)/\gamma_1, \quad (2.10)$$

this taking account of the relative magnitudes of the various material parameters and the likely magnetic field strength and value of wavenumber q. Similarly for the twist-bend mode the same approximations lead to

$$\omega_1^{(2)} = -ik(\phi)(q^2 h(\phi) + \chi_a H^2)/\gamma_1 \tilde{k}(\phi),$$

$$\omega_2^{(2)} = -iq^2 \tilde{k}(\phi)/\rho, \quad (2.11)$$

where

$$\tilde{k}(\phi) = k(\phi) - \alpha_2^2 \sin^2 \phi/\gamma_1. \quad (2.12)$$

In each mode, the first decay rate is very much slower than the second, and it is this slower rate which is measured in light scattering experiments.

If the dielectric anisotropy ϵ_a is positive, and an electric field E replaces the magnetic field, the above analysis immediately suggests observable decay rates $\Lambda^{(\alpha)}$ of the form

$$\Lambda^{(1)} = i\omega_1^{(1)} = g(\phi)(q^2 f(\phi) + \epsilon_a E^2)/\gamma_1 \tilde{g}(\phi),$$

$$\Lambda^{(2)} = i\omega_1^{(2)} = k(\phi)(q^2 h(\phi) + \epsilon_a E^2)/\gamma_1 \tilde{k}(\phi). \quad (2.13)$$

Alternatively, rewriting these in terms of the applied voltage V, one has

$$\Lambda^{(\alpha)} = A^{(\alpha)} + M^{(\alpha)} V^2, \qquad \alpha = 1,2, \quad (2.14)$$

where

$$A^{(1)} = q^2 g(\phi) f(\phi)/\gamma_1 \bar{g}(\phi), \qquad M^{(1)} = \epsilon_a g(\phi)/\gamma_1 \bar{g}(\phi) d^2,$$

$$A^{(2)} = q^2 k(\phi) h(\phi)/\gamma_1 \bar{k}(\phi), \qquad M^{(2)} = \epsilon_a k(\phi)/\gamma_1 \bar{k}(\phi) d^2, \qquad (2.15)$$

and d denotes the cell thickness. If for an electric field one employs analogous expressions to those for a magnetic field, there is of course the question as to whether the fact that the dielectric anisotropy is large compared with the diamagnetic susceptibility affects the issue. For small perturbations of a uniformly aligned nematic, however, one might initially anticipate that the above are valid.

3. EXPERIMENTAL DETAILS

Figure 2 shows schematically the experimental layout. Light from a 1 mW He/Ne laser passes through a rotatable polariser, used to select the appropriate mode, and is focused into a cylindrical sample cell holder that contains the sealed sample immersed in an index-matching and thermo-regulated bath (to within ± 0.1°C). The light scattered from the sample then passes through an analyser, polarised in the scattering plane, and is focused through a pin-hole onto the detector of a photomultiplier (Malvern Instruments RR 127). By positioning the photomultiplier on the arm of a rotatable spectrometer table, it was possible to achieve servo angular control of the scattering angle.

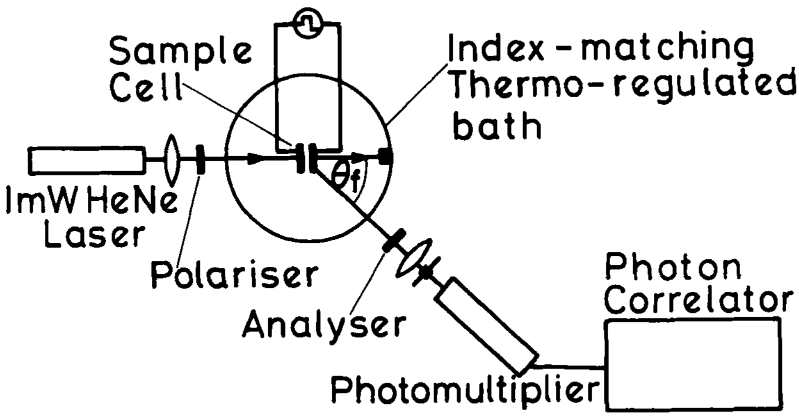


FIGURE 2 Schematic representation of the experimental arrangement.

The sample cell was constructed from ITO coated glass substrates treated with lecithin for homeotropic alignment, and separated by a 50 μm Mylar spacer and a plastic seal.

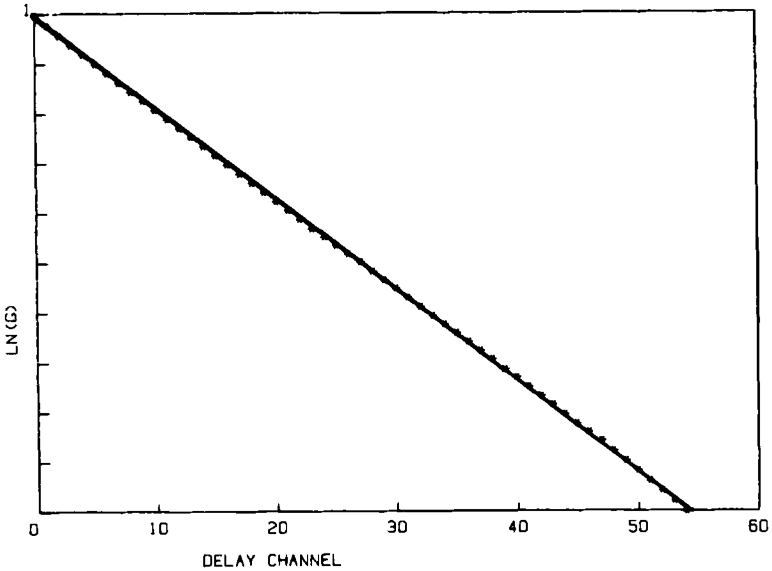
A computer controlled 64 channel photon correlator (Malvern Instruments K 7027 'Loglin' correlator) analyses the scattered light and determines the second order autocorrelation function. For a single Lorentzian spectrum, the normalised value of this autocorrelation function can be fitted to a semi-log plot, whose gradient gives the decay rate of the alignment (director) fluctuations directly. As a measure of the Lorentzian nature of the scattering spectrum, we use the method of cumulant expansion proposed by Pusey.[8] By fitting both linear and quadratic curves to the semi-log plot of the normalised correlogram, one obtains a direct estimate of the variance of the distribution called the Pusey Q factor, or polydispersity of the distribution. For a good single exponential decay Q is typically less than 0·05.

The spectrum of scattered light was measured using a pure homodyne technique over a range of scattering angles (10°–30°), and the effect of applying an *ac* squarewave electric field corresponding to up to 50 volts RMS and over a range of frequencies from 50 to 10^4 Hz investigated for both scattering modes. Because the scattering in the splay-bend mode is polarised, care was taken to eliminate any stray scattering which might give rise to a heterodyne component in the spectrum. To check for the presence of stray scattering, we heated the sample above its nematic to isotropic transition temperature (35·3°C) and this reduced the count-rate to about 1% of the signal in the nematic phase, confirming the pure homodyne nature of the scattered spectrum.

4. RESULTS

For the twist-bend mode the spectra obtained in the presence of an *ac* squarewave electric field were Lorentzian, fitting well to a single exponential decay rate (see Figure 3a). This decay rate increases linearly with the square of the applied voltage (see Figure 4) and is independent of the frequency as predicted above (equations (2.14) and (2.15)). The slope and intercept of these straight line plots for different scattering angles allow one to calculate the twist elastic constant K_2 and twist viscosity γ_1 (see Figures 5 and 6). This required the measurement of the cell spacing (by an interferometric technique), the dielectric anisotropy (by capacitance method), and the

(a)



(b)

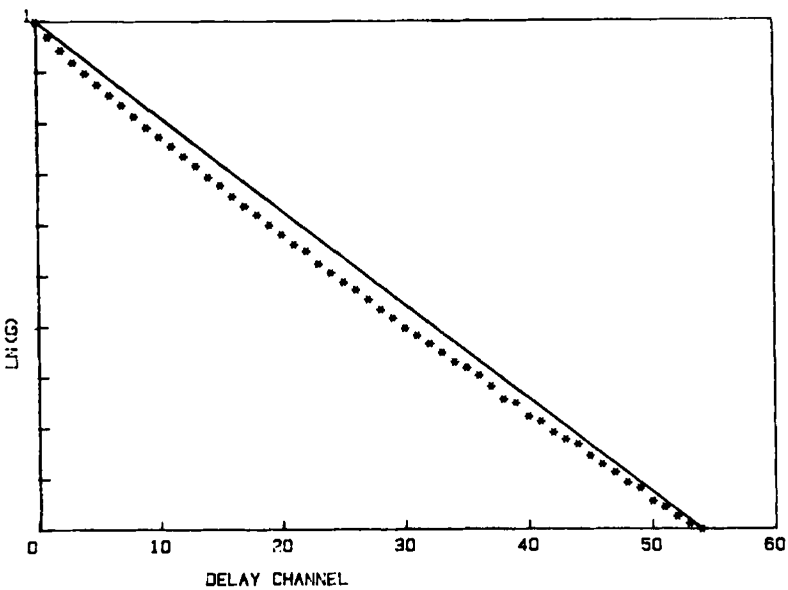


FIGURE 3 Typical semi-log plot of the normalised autocorrelation function for (a) the twist-bend mode showing Lorentzian behaviour, and (b) the splay-bend mode showing non-Lorentzian behaviour.

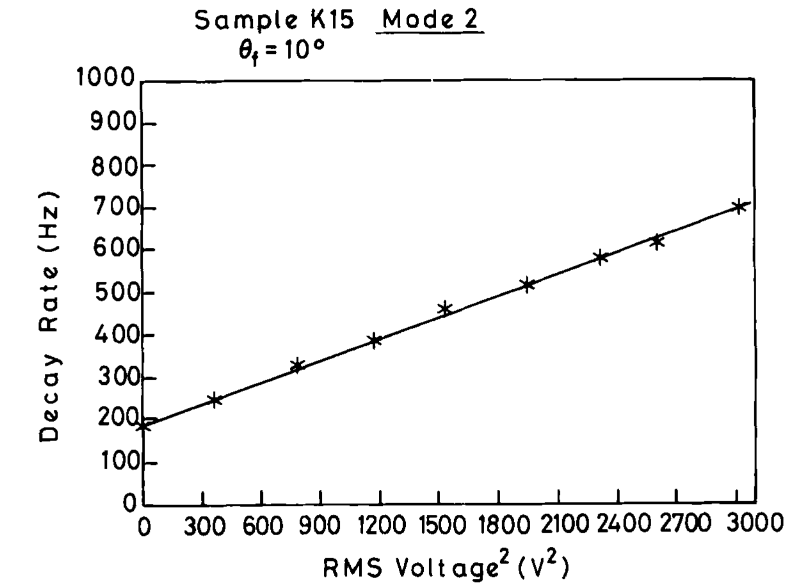


FIGURE 4 Typical plot of decay rate against the square of the RMS voltage for the twist-bend mode.

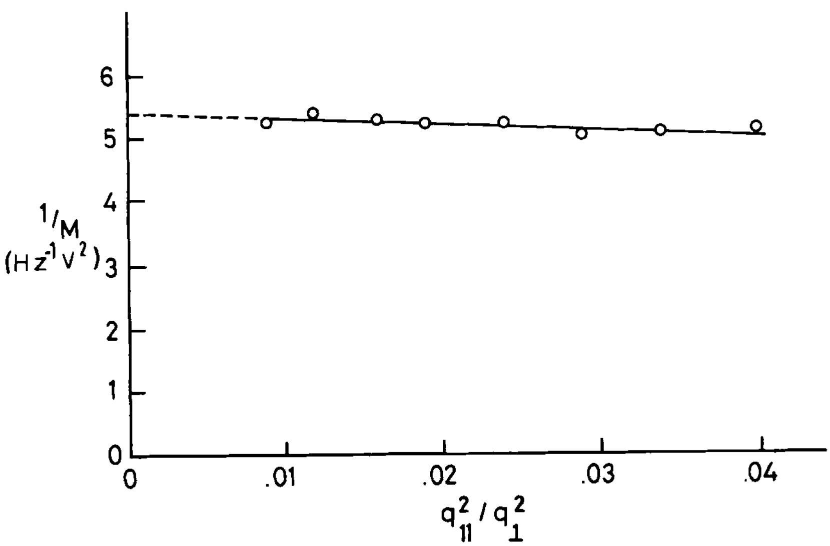


FIGURE 5 Plot of the reciprocal slope 1/M from Figure 4 against $q_{\parallel}^2/q_{\perp}^2$ The intercept gives a value of γ_1 equal to 121 *cP* for 5CB at 20°C.

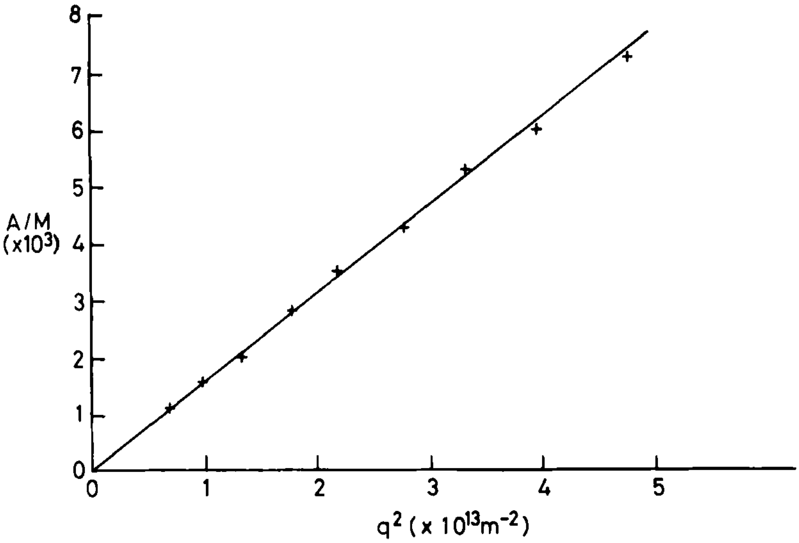


FIGURE 6 Plot of the ratio of intercept to slope (A/M) from Figure 4 against q^2. The slope gives a value of K_2 equal to $3 \cdot 6 \times 10^{-12}$ N for 5CB at 20°C.

correction of the scattering vector using the refractive indices (Abbé refractometer).

For 5CB at 20°C, we find that

$$K_2 = (3 \cdot 6 \pm 0 \cdot 3) \times 10^{-12} N, \qquad \gamma_1 = 121 \pm 6\, cP \qquad (4.1)$$

in good agreement with existing literature values.[9,10]

For the splay-bend mode, however, the application of an *ac* square-wave field led to non-Lorentzian scattering spectra (see Figure 3b). The deviation from a single exponential decay rate occurred over the entire range of scattering angles investigated (10°–30°), being most noticeable for reasonably large field strengths (corresponding to more than 20V) and at small scattering angles where the field dominates the decay rate. The resulting spectra and the intensity of the scattered light show a remarkable dependence upon the frequency of the applied field. Figure 7 illustrates typical plots of the Q factor against the frequency of the applied field for different scattering angles. A minimum occurs in this Q factor indicating essentially a single exponential decay rate over a narrow range of frequency. The frequency at which this minimum occurs is approximately half the measured

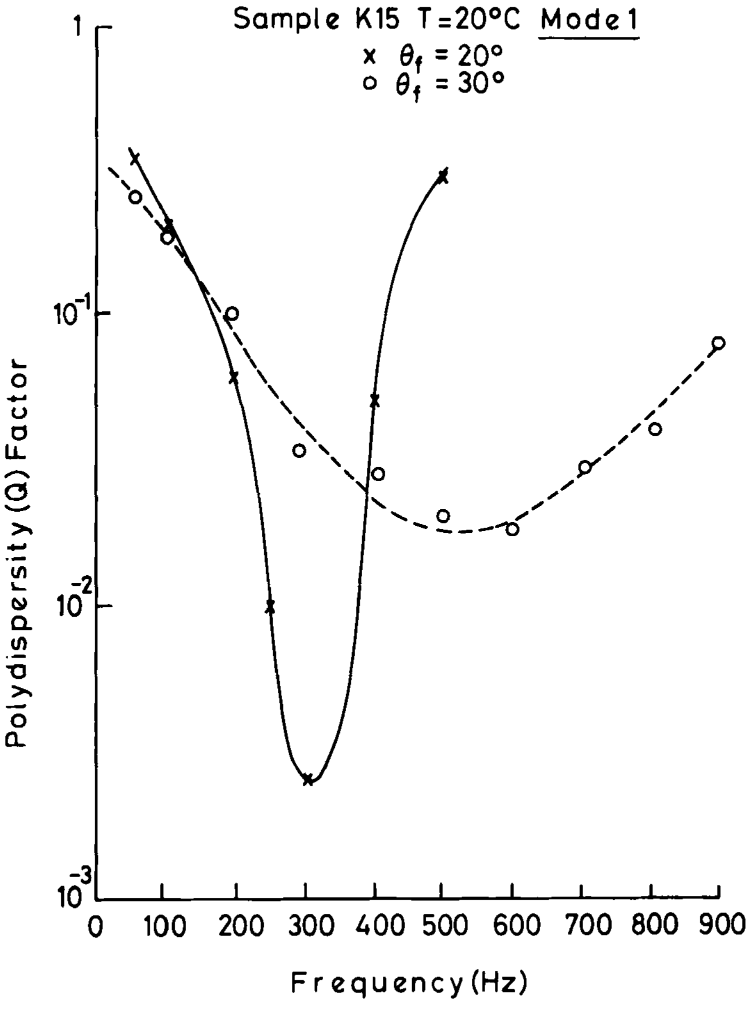


FIGURE 7 Two plots of the Pusey polydispersity factor against frequency for the splay-bend mode for 5CB (K15) at 20°C.

decay rate. Non-Lorentzian behaviour persisted up to the highest frequencies employed (10^4 Hz).

One possible explanation of the above unexpected behaviour is that some instability is occurring in the sample. Consequently the sample was examined under a polarising microscope, but no instability was detectable over the range of fields used.

5. THEORY OF FLUCTUATIONS IN AN ELECTRIC FIELD

In this section we attempt to explain the above observations by considering in greater detail perturbations to a uniformly aligned nematic in the presence of a stabilising electric field. Given the frequency dependence of the non-Lorentzian decay rate in an *ac* squarewave field described above, flexoelectric and conduction effects involving a linear rather than quadratic dependence upon the field quickly come to mind as possible explanations of this unexpected behaviour. Consequently we investigate the influence of both factors, seeking solutions of Maxwell's equation in a restricted form as well as the continuum equations for liquid crystals. However, it is convenient for the present to confine the analysis to the case of a *dc* electric field.

Let E^0 denote the undisturbed uniform electric field strength, and in addition to the small disturbances to alignment and flow given by (2.1) and (2.2) consider a small perturbation to the electric field $\mathbf{E}$ of the form

$$\mathbf{E} = E^0\mathbf{n}^0 + \hat{\mathbf{E}} \exp i(\mathbf{q} \cdot \mathbf{x} - \omega t) \tag{5.1}$$

$\hat{\mathbf{E}}$ being a constant vector. Also, it is necessary to include a free charge distribution e given by

$$e = \hat{e} \exp i(\mathbf{q} \cdot \mathbf{x} - \omega t) \tag{5.2}$$

where $\hat{e}$ is a constant. These variables are subject to Maxwell's equations and for the present problem we assume that it suffices to consider a curtailed version of these,

$$\operatorname{curl} \mathbf{E} = 0, \quad \operatorname{div} \mathbf{D} = 4\pi e, \quad \dot{e} + \operatorname{div} \mathbf{J} = 0 \tag{5.3}$$

the superposed dot denoting a time derivative. The displacement vector $\mathbf{D}$ and the electric current density vector $\mathbf{J}$ are related to $\mathbf{E}$ by

$$\mathbf{D} = \epsilon_\perp \mathbf{E} + \epsilon_a(\mathbf{n} \cdot \mathbf{E})\mathbf{n} + e_1(\operatorname{div} \mathbf{n})\mathbf{n} + e_3(\mathbf{n} \cdot \operatorname{grad})\mathbf{n},$$

$$\epsilon_a = \epsilon_\parallel - \epsilon_\perp, \qquad \mathbf{J} = \sigma_\perp \mathbf{E} + \sigma_a(\mathbf{n} \cdot \mathbf{E})\mathbf{n}, \qquad \sigma_a = \sigma_\parallel - \sigma_\perp, \tag{5.4}$$

the ϵ's, e's and σ's denoting dielectric permittivities, flexoelectric coefficients and conductivities, respectively.

Equation (5.1) and the first of equations (5.3) quickly lead one to the result

$$\hat{\mathbf{E}} = \hat{E}\mathbf{q}, \tag{5.5}$$

and again ignoring products of perturbations, the second of equations (5.3) with the first of (5.4) yields

$$iq^2\epsilon(\phi)\hat{E} = 4\pi\hat{e} - i(\epsilon_a E^0 + iqe(\phi))\hat{\mathbf{n}} \cdot \mathbf{q}, \tag{5.6}$$

with

$$\epsilon(\phi) = \epsilon_\perp + \epsilon_a \sin^2\phi, \qquad e(\phi) = (e_1 + e_3)\sin\phi, \tag{5.7}$$

this last equation giving the pertubation to the electric field in terms of the other unknowns. The last of equations (5.3) reduces to

$$(\omega\epsilon(\phi) + 4\pi i\sigma(\phi))\hat{e} + (\beta E^0 + iq\sigma(\phi)e(\phi))\hat{\mathbf{n}} \cdot \mathbf{q} = 0, \tag{5.8}$$

where

$$\sigma(\phi) = \sigma_\perp + \sigma_a \sin^2\phi, \qquad \beta = \epsilon_a\sigma_\perp - \sigma_a\epsilon_\perp. \tag{5.9}$$

From equations (5.6) and (5.8) it is immediately clear that the perturbation to the electric field and the free charge distribution are coupled to the splay-bend mode, but not to the twist-bend mode.

The relevant continuum equations in Cartesian tensor notation are

$$\rho\dot{v}_i = eE_i - \tilde{p}_{,i} + \tilde{g}_k n_{k,i} + \tilde{t}_{ij,j},$$

$$\left(\frac{\partial\mathcal{E}}{\partial n_{i,j}}\right)_{,j} - \frac{\partial\mathcal{E}}{\partial n_i} + \tilde{g}_i + \gamma n_i = 0, \tag{5.10}$$

with

$$\tilde{p} = p + \mathcal{E}, \qquad \mathcal{E} = W - W_e - W_f \tag{5.11}$$

where p denotes pressure, W the Frank-Oseen elastic energy, W_e the electric and W_f the flexoelectric contributions, and $\tilde{\mathbf{t}}$ and $\tilde{\mathbf{g}}$ the dissipative terms in the stress tensor and intrinsic generalised body force, respectively.[6,11] Proceeding as in section 2 one finds for the splay-bend mode

$$(\rho\omega + iq^2 g(\phi))v_1 + iq\omega m(\phi)n_1 - iE^0\hat{e} = 0,$$

$$iqm(\phi)v_1 + (i\omega\gamma_1 - q^2 f(\phi) - \epsilon_a\epsilon_\parallel E^{02}/\epsilon(\phi)$$

$$- q^2 e^2(\phi)\cos^2\phi/\epsilon(\phi))n_1 + (4\pi/\epsilon(\phi))(\epsilon_a E^0/iq - e(\phi))\hat{e} = 0, \quad (5.12)$$

and for the twist-bend mode

$$(\rho\omega + iq^2 k(\phi))v_2 - iq\omega\alpha_2 \sin\phi n_2 = 0,$$

$$iq\alpha_2 \sin\phi v_2 - (i\omega\gamma_1 - q^2 h(\phi) - \epsilon_a E^{02})n_2 = 0. \quad (5.13)$$

Consequently in agreement with our observations these additional factors do not influence the latter mode, but only the former.

At this point it is of interest to compare the relative importance of the flexoelectric and conduction contributions to the splay-bend mode equations. If one neglects conduction entirely, equation (5.8) implies that free charges are absent and therefore that the last terms in equations (5.12) are zero. Consequently, the flexoelectric contribution in the absence of conduction leads to a single term in equations (5.12). This is negligible compared with the torque arising from the external field if

$$e^2(\phi) << \epsilon_a^2 E^{02}/q^2. \quad (5.14)$$

Furthermore, if this last condition holds, it follows from the above equations that all flexoelectric terms are negligible even when conduction is included. Values available in the literature for the flexoelectric coefficients[12-14] suggest that the condition (5.14) is satisfied in the experiments described above, and therefore in the subsequent discussion we omit the flexoelectric terms.

Before proceeding it is of interest to note that the electric field contribution when one omits *both* conduction and flexoelectric terms in these equations is not analogous to the corresponding magnetic field term in equations (2.5). This difference is a consequence of allowing a perturbation in the electric field, which with hindsight should have been included in the original analysis at the end of section 2.

Confining attention to conduction effects, the equations for the splay-bend mode are

$$(\rho\omega + iq^2 g(\phi))v_1 + iq\omega m(\phi)n_1 - iE^0\hat{e} = 0,$$

$$iqm(\phi)v_1 + (i\omega\gamma_1 - q^2 f(\phi) - \epsilon_a\epsilon_\parallel E^{02}/\epsilon(\phi))n_1 + 4\pi\epsilon_a E^0\hat{e}/iq\epsilon(\phi) = 0,$$

$$q\beta\cos^2\phi E^0 n_1 + (\omega\epsilon(\phi) + 4\pi i\sigma(\phi))\hat{e} = 0, \tag{5.15}$$

and these lead to the following equation for ω

$$\begin{vmatrix} \rho\xi + g(\phi) & -\xi m(\phi) & -1 \\ -m(\phi) & f(\phi) + \gamma_1\xi + \epsilon_a\epsilon_\parallel E^{02}/q^2\epsilon(\phi) & 4\pi\epsilon_a/\epsilon(\phi) \\ 0 & \beta\cos^2\phi E^{02}/q^2 & q^2\xi\epsilon(\phi) + 4\pi\sigma(\phi) \end{vmatrix} = 0 \tag{5.16}$$

where we have set

$$q^2\xi = -i\omega. \tag{5.17}$$

This determinantal equation yields a cubic

$$\xi^3 + a\xi^2 + b\xi + c = 0 \tag{5.18}$$

where the coefficients are given by

$$\left.\begin{aligned} a &= \tilde{G} + K + F + P, \\ b &= G(K + F) + P(\tilde{G} + F) + KQ, \\ c &= GKQ + ML(P - Q) + GFP, \\ &\text{with} \\ G &= g(\phi)/\rho, \quad \tilde{G} = \tilde{g}(\phi)/\rho, \quad M = m(\phi)/\rho. \\ K &= \epsilon_a\epsilon_\parallel E^{02}/\gamma_1 q^2\epsilon(\phi), \quad L = \epsilon_\parallel E^{02}/4\pi\gamma_1 q^2, \\ F &= f(\phi)/\gamma_1, \quad P = 4\pi\sigma(\phi)/q^2\epsilon(\phi), \quad Q = 4\pi\sigma_\parallel/q^2\epsilon_\parallel. \end{aligned}\right\} \tag{5.19}$$

These coefficients differ significantly in magnitude, and we estimate that for the conditions prevailing in the above experiments

$$a \sim 0(1), \qquad b \sim 0(10^{-4}), \qquad c \sim 0(10^{-9}) \qquad (5.20)$$

in cgs units.

If one derives the reduced form of equation (5.18) and examines its discriminant, it is possible to show that the three roots are real and unequal when the coefficients satisfy (5.20). Their values follow from Cardan's formula[15] and to a good approximation are

$$\xi_1 = -a = -\bar{g}(\phi)/\rho,$$

$$\xi_2 = -b/a = -\epsilon_a\epsilon_\parallel E^{0^2} g(\phi)/\gamma_1 q^2 \epsilon(\phi)\bar{g}(\phi),$$

$$\xi_3 = -c/b = -4\pi\sigma_\parallel/q^2\epsilon_\parallel - \beta m(\phi)\cos^2\phi/q^2\epsilon_a\epsilon_\parallel g(\phi). \qquad (5.21)$$

The first two roots are basically the counterparts of the results (2.9) and the third arises from conduction effects. While the latter corresponds to the slowest decay rate, the difference in magnitude between ξ_2 and ξ_3 is not so large, and presumably both decay rates would contribute to light scattering measurements.

6. CONCLUDING REMARKS

Our interest in light scattering from a nematic in an electric field stems from its attractions as a means of measuring viscosity coefficients for these materials. In particular one can employ small samples in which the alignment is readily controlled with the possibility of monitoring behaviour optically. Another advantage lies in the use of an electric field which yields sufficiently large field strengths more readily than a magnetic field. Also the related theory developed by the Orsay Group appears sound, having been well tested. Disagreement between our experimental observations and this theory was therefore surprising and clearly requires explanation before one can trust data from such measurements.

While the analysis of the previous section does not correspond exactly to our experiments in that it assumes a *dc* rather than *ac* field, it does offer a likely explanation of the source of discrepancy between existing theory and our experimental results for the splay-bend mode. In view of our conclusions for a *dc* field, it does not appear unreasonable to assume that conduction effects give rise to a further decay rate not so different in magnitude from that usually observed in light scattering experiments, and that the observed non-Lorentzian behaviour therefore stems from this source. In addition flexoelectric effects could conceivably complicate the response of this mode, although this does not appear to be the case at present. Lastly one must add that the agreement between the preceding theory and our observations that the twist-bend mode is unaffected by these additional complications lends strong support to the above explanation.

Granted that conduction and possibly also flexoelectric effects are the cause of the non-Lorentzian behaviour of the splay-bend mode in the presence of an electric field, it seems that little can be obtained from such measurements given the complexity of the phenomenon. However, one can use an electric field in light scattering experiments to obtain data rather readily from the twist-bend mode, and of course measurements of the splay-bend mode in the absence of an electric field provide further data.

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